ASTRONOMICAL TABLES FOR PISA

IN MS. KØBENHAVN K.B., GKS 277, FOL.[1]

By Fritz S. Pedersen

A set of astronomical tables for Pisa, composed about AD 1143 by Abraham b. Ezra, has so far been known from five manuscripts. This paper presents a sixth one, from the mid-thirteenth century, noticed in 1994 by the late Wilbur Knorr in the Royal Library, Copenhagen, and first identified on the present occasion. The mean motion tables are likely to have ended in 1209 originally, and they have had various continuations in the different witnesses. The Copenhagen manuscript, too, has been continued, no doubt by two different persons, one of whom seems to have wanted to adapt the tables for use in Paris, whereas the other was a particularly inept calculator. This seems to be quite a typical state of affairs when complicated arrays of numerals were to be updated.

(1) The manuscript K.

København K.B., GkS 277 fol. (from now on denoted "K") is a composite manuscript in Latin, in several hands, all apparently from about the mid 13th century. It has been described and inventarized by Ellen Jørgensen.[2] It contains upwards of 20 treatises, mainly concerning arithmetic and geometry, of widely varying ages; some of the more recent ones are Sacrobosco's *Compotus*, from 1232-35,[3] and his *Sphere*, undated.

The astronomical tables to be treated here are on ff. 183v-190v. They are accompanied by a set of rules beginning on 190va and ending abruptly at 193vb, which is the end of the book. One or more leaves have been lost, though not much of the text need be missing.[4]

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[1] Many thanks to Prof. Raymond P. Mercier, to Mr. Martin Røpcke, and to the staff of Håndskriftafdelingen, Det kongelige Bibliotek, Copenhagen, for their kind help during the preparation of this paper. Any errors have been made by me, not by them.

[2] Jørgensen 1926, 417-418. I shall refer to the medieval foliation in the manuscript, as does Jørgensen's catalogue, rather than to the modern one.

[3] See O. Pedersen 1985, 185.

[4] On the basis of a summary inspection that is certainly subject to revision, it appears that ff. 183-190 form a quire; that f. 191 is single and has been attached to this quire; and

This manuscript was brought to my attention as early as in August 1994, by the late Wilbur Knorr, who examined it during a visit in Copenhagen. For any number of reasons the case has stayed dormant until recently, but as it is quite a curious example of medieval philology, I am happy to offer this account as a greeting to Karsten Friis-Jensen, though the field is very different indeed from his.

(2) Place and date of the tables and rules in ms. K.

The tables and rules are in one hand, which has also written a few glosses in other parts of the book. These include one note on 91v-92r mentioning “nunc scilicet anno domini M'o CC XL” and “hodie autem, scilicet M'o CC XL gratie anno”, dating the hand to probably the 1240s.[5] On 180r, a note in the same hand states that a table on the page is valid “in climate septimo”, the geographical zone that includes northern France and Paris.

Some further indications of place and date can be had from the tables and rules. On 183v, on the first page of the tables, there are 14 rubricated verses beginning,

> Accipe Pisani doctoris dogma decorum,
> Quod super hac arte dedit hortatu sociorum…[6]

In the rules we have the passages,

> (191va-b) …longitudo Parisiensis est 33 graduum sicut Pisana longitudo, quod per eclipses probatum est, habent ergo eandem meridiem Parisius et Pisa…
>
> (192va) (rubr.) De adequatione planetarum. (text.) Sciendum quod tabule iste sunt composite secundum clima Pisanum et translate secundum clima Parisiense, et habent in se motum octave inclusum … (192vb) A medio motu planete subtrahe augem eius, qui aux crescit omni anno 51 secundis. Anno autem domini M'o CC XLVIII fuit aux solis 2 signa 28 minuta 20 secunda[7]…

This, together with the dating 1254 in the apogee table to be mentioned in (3,c) below, indicates that the present copy of the tables and rules is from about 1250 and is meant to be valid for Paris. The tables are stated to have been for Pisa originally. The question is, then, whether they have parallels in

that ff. 192-193 form a bifolium of their own. No text is missing or displaced on this stretch. Thus perhaps just one leaf, originally belonging with f. 191, is missing.

[5] This note was seen, and the dating exploited, by O. Pedersen 1985, 183 with note 52.

[6] The remaining 12 verses have no factual content. They do cull flowers from Ovid (*Ex Ponto* 1,1,1, “Naso Tomitanae…”) and Boëthius (*Cons. Phil.* 3, metrum 9, “O qui perpetua…”).

[7] Should be 2 signs (=60°) + 28°20', cf. (3,c) below.

other copies of tables for Pisa. This turns out to be the case, as will be seen shortly. But before that, a brief description of the tables and rules of K is in order.

(3) Structure of the tables and rules in ms. K.

The tables are those necessary for computing the positions of the Sun, the Moon, the lunar node and the five planets on the ecliptic at any given time according to the Julian calendar. In each of these eight cases (except the node) there are two kinds of tables, namely,

(a) *mean motion tables* (see the samples in (5-6) below). These are tables starting at a specified time (the "radix") with a specified value (the "radix value"), which indicates the mean position on the ecliptic (in the case of the "mean motus" tables) or on the epicycle (in the case of the "argument" tables), and they increase linearly with time from this value, according to the mean velocity of the object in question. Roughly, the mean velocity is the angular velocity the object would have if it moved evenly on the ecliptic (or the epicycle). For each object, the mean motus table is in six parts, namely, for "collected years" (years taken 20 at a time, starting from the radix), for "expanded years" (years taken one at a time, from 1 to 20), and for months, days, hours, and minutes of an hour. To get the mean position at any given time, consider this time as a sum of collected years, single years, months, etc., according to the parts of the mean motion table; then enter each part with the appropriate component, and finally add together all the tabular values obtained.

In the collected-year tables of K, the radix is "AD 1169" elapsed, and the year begins in March. In effect, then, "AD 1169" ends in February of the year that would be 1170 in the modern calendar, two years before the first February that contains a leap-day. Accordingly, in the expanded-year table, year 1 is considered a normal year of 365 days, whereas year 2 and every fourth year following this are considered leap years of 366 days.

There are two mean motion tables for the Moon, so in total there are nine mean motion tables.

(b) *Equation tables.* These are meant for correcting the mean positions, obtained from the mean motion tables, according to the eccentricity of the deferent and to the object's position on the epicycle. In this way, the true position on the ecliptic is obtained.

The equation tables in ms. K are quite like those which appear in the Toledan Tables, and similar to the source of the latter, namely, those of Battani.[8] Only in the case of the Sun do they show special values.[9]

There is no equation table for the lunar node, since this is supposed to revolve uniformly. This makes for a total of seven equation tables.

(c) One will also need a set of values for the places of the planetary *apogees* ("auges"). There is a table of such values on f. 183rb, dated AD 1254, in the same hand as the main set of tables. The value for the solar apogee is 2 signs + 28°24'.

The rules in K, insofar as they have been preserved (see (1) above), consist of three parts plus some lesser chapters, as follows:

(d) 190va-191rb: On reckoning with sexagesimal fractions ("minutie philosophice"). This seems to be a paraphrase of a text known in a few copies from elsewhere.[10]

(e) 191rb-192va: On making tables. This section deals with quite a lot of different tables apart from those mentioned. The instructions seem rather inept and may serve for recognizing the tables, but hardly for emending them, not to speak of constructing them afresh.[11]

(f) 192va-193va: On finding the planets' positions. This section is the one relevant for the tables in our manuscript. The rules are phrased much like the common canons for the Toledan tables, but are not the same as these.[12]

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[8] Toledan Tables: Toomer 1968, 56 ff.; F.S. Pedersen 2002, 1245 ff. Battani: Nallino 1899+, Vol. II, 78ff., 108ff. As in the Toledan Tables, the sub-tables for stations of the five planets are incorporated in the equation tables, whereas they constitute a separate table in Battani.

[9] (183v-184r:) maximum equation ("directio") of Sun, 1;59,8° at 3s0°-3° (against 1;59,10° at 3s(1°-)2° in the other sources); maximum declination of Sun, 23;33,8° at 3s0°-1° (against different values in the other sources). In the table of lunar latitude (185r), the maximum latitude appears to be 4;59,57° for 3s0°, instead of the usual 5;0,0°; this error, along with an attempt to adapt the values on the interval 2s26°-29°, is shared with mss. Vv Vv2, for which see (4) below. The rest of the table, however, has the Toledan values.

[10] (rubr.:) Qualiter algorismo sit utendum in negotio tabularum istarum. (inc.:) Quoniam opus tabularum istarum (et) philosophicarum minutiarum sine algorismo elabor<ar>i non potest. Similar incipits are in Thorndike & Kibre 1963, 1292, and in the Jordanus database <jordanus.ign.uni-muenchen.de>, reported from: Laon BV 425, 13th c., 2rb-2vb; Lüneburg Ratsbücherei Misc. D 2° 11, 14th c., 68vb ff.; Paris BN 7197, 15th c., 30r-31r. I have compared the text in K with Laon 425, and have found that they have significant passages in common, but that one or both are paraphrased to some extent.

[11] (rubr.:) Doctrina qualiter componende sint tabule. (inc.:) De compositionibus tabularum dicturi primo compositiones medii motus solis dicemus.

[12] (rubr.:) De adequatione planetarum. (inc.:) Sciendum quod tabule iste sunt composite secundum clima Pisanum. More text, see (2) above. The incipits of items (e) and (f) are not

(g) 193va-b: Lesser rules for stations and retrogradations, for conjunctions and oppositions of the Sun and Moon, and for eclipses. The text ends abruptly.

Since the equation tables are mainly the same as the common ones, and the rules are rather nondescript and difficult to identify, one may choose to pay special attention to the mean motion tables, and this will be done in the following. Such tables are in any case those characteristic of a table collection, being adapted to the longitude of the place in question.

(4) The Pisa mean motion tables in other manuscripts.

Mean motion tables with values comparable to those in ms. K are known from five other manuscripts, namely,

Mf Madrid B.N., 10009, 1r, 13th c.
(only Saturn, mean motus, down to hours)
\- Oxford Bodl.L., Selden sup. 90, 30-34
Py Paris B.N., lat. 16207, 19r-36r, 13th c.
Pz Paris B.N., lat. 16208, 4r-v, 12th-13th c.[13]
Vv Vatican City, B.A.V., Vat. lat. 3119, 1v-9v, 19r-42r, 12th-13th c.; (Vv2:) 10r-18v, 13th c. (extra copy of lunar mean motus, lunar argument and solar and lunar equations)

None of these manuscripts show indications of the origin of the tables. However, some of their characteristic features (notably, the entrances in the collected-year tables and the placement of the leap years) are referenced in three texts found elsewhere.[14] These texts make it plain that the tables are those for Pisa composed, perhaps in 1143, by the Jewish traveller and polymath Abraham b. Ezra, who was active in the region during the years 1141-

found in the usual catalogues, and the texts seem to show no specific resemblance to the three Abraham b. Ezra texts cited in section (4) below. Texts of the canons for the Toledan tables: F.S.Pedersen 2002, Parts 1-2.

[13] This and the three preceding are cited by Mercier 2004, 28-29. Cambr. U.L., Add. 6866, 150v contains a solar apogee table that is probably irrelevant in the present connection; Prof. Mercier kindly did me the favour of clearing this up. Berlin lat. fol. 307, ff. 27-28, 30-32, quoted by Birkenmajer 1919, 151, may contain relevant matter, but I have not seen it; some details on it are given by Millás 1947, 59-60.

[14] The texts, as examined by Mercier 2004, 13-17, 28-29, are: (1) Abraham b. Ezra, "Cognitum est corpus solare", printed by Millás 1947; (2) anonymous (probably a Christian), "Ptolomeus et multi sapientum", Cambr. Fitzw. Mus., McClean 165, 67r-76v, sub-incipit "Tabulas compositurus hoc ordine procedes" on 68v; (3) Abraham, "Volens in tabulis operari", London B.L., Arundel 377, 56va-63ra, for which cf. also Millás 1947, 60-65.

45.[15] For details see Mercier 2004. The texts state that the longitude of Pisa is 33° from the West (as does ms. K, see (2) above), and that the mean solar velocity is from al-Sûfî (d. AD 986). Mercier (2004, 10-12) makes the case that our tables are indeed based on those of al-Sûfî, and that the latter were originally calculated for Baghdad, with the epoch of Hijra year 370 elapsed, or about AD 981.

I have seen films of the manuscripts Mf Py Pz Vv Vv2. Their collected-year tables cover various ranges, but those of Py Pz start in "1169" as do those of K. The ranges are: 1209-1229 Mf; 1169-1309 Py; 1169-1289 Pz; 1029-1209 Vv, with continuations for 1229-1289 or -1309 (in one case, -1269) by a later hand. The two mean motion tables in Vv2 show the ranges 1009- or 1029-1209, with continuations for 1229-1309 or -1289 in a later hand.

Other tables: Vv (Vv2) Py contain planetary equation tables. These are normal ones, of the same kind as in K, but Vv Vv2 show the same peculiar tables for the Sun.[16] Thus perhaps the solar equation table has the same origin as the Pisa mean motion tables.

Pz has an extra mean motion table for the apogee of the Sun "ad meridiem Londoniarum", for 1169-1289, with radix said to be 1149 (no tabular value given; that for 1249 is 2 signs + 27°33'18".). The velocity is 54"-55" per year, thus the common precession rate of 1° per 66 years. No doubt this belongs to the London tables, which are related to the Pisan ones.

(5) Samples of the mean motion tables in the common tradition.

The following is not an edition, but a sample of the mean motion tables in a form convenient for comparison with K. Thus, for the collected-year tables, I record the entries labelled "1169"-"1249", corresponding to those which occur in K; samples of the subtables for expanded years and of days are also shown, but only enough to enable the reader to follow the checks for internal consistency carried out in the sections entitled "Validation". I mostly try to show the readings that are closest to the source for K; to do this, I have had to be quite eclectic.

The values that appear in the tables are expressed in signs (each of 30 degrees), degrees, minutes and seconds, for instance: <9 1 57 39>. According to circumstances, this will be quoted as 9s1°57'39", or 271°57'39", or 271;57,39°.

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[15] There is some doubt as to whether the tables are Abraham's own or a revision of them, cf. Millás 1947, 65 and Mercier 2004, 3-4. This does not affect the present argument.

[16] The maximum solar declination of 23;33,8° for 3s0°-1° is shown by Vv2, whereas Py Vv lack this table. The maximum solar equation of 1;59,8° for 3s0°-3° is shown by Vv Vv2; Py has the Toledan values, with max. 1;59,10° for 3s1°-3°.

In the expanded-year table, years 2,6,…,18 are taken to be leap-years, so that they increment the tabular values by the motion in 366 days; the rest of the years are normal years, incrementing the values by the motion in 365 days. The value for 365 days is that shown for year 1 in the expanded-year table, and that for 366 days can be had by adding the 1-day value also shown. Unless otherwise noted, the table-maker has ignored any sexagesimals beyond seconds. This, to be sure, affects the precision of the tables and of the parameters that can be extracted from them.

In the collected-year table the increment is constant. Normally it is equal to the 20-year value from the expanded-year table; if not, one may suppose that it has been calculated independently, or that the expanded-year table is imprecise or corrupt.

(5.1) Sun, mean motus.

Py1 = Py,19r. Py2 = Py,20v. Py = Py1 & Py2.

Collected years, sample with the range of K:

	s	°	'	"	
1169	11	15	33	20	
1189	11	15	42	20	
1209	11	15	51	20	
1229	11	16	0	20	15° Py1
1249	11	16	9	20	

Expanded years, sample for years 1-3 and 19-20:

	s	°	'	"	
1	11	29	45	40	
2	0	0	30	28	
3	0	0	16	8	
19	0	0	23	20	
20	0	0	9	0	

	s	°	'	"	
1 day	0	0	59	8	*def.* Py1

Validation: These tables are regular. The increments in the expanded-year table (359;45,40° for 365 days, 0;44,48° for 366 days) are applied consistently, and yield the 20-year value shown. This is also consistently used in the collected-year table.

The 1-year value of 359;45,40° implies a year of 365 days * 360 / 359;45,40 = 365;14,32… days = 365.2424 days, which is a tropical year of just about 365 + 1/4 – 1/131 days. This illustrates the statement, under (1) above, that the "motus octave" is included in the velocities.

(5.2) Moon, mean motus.

Vv = Vv,7r. Vv2 = Vv,13r (both m2 for 1229-1309).

1169	4	10	19	53	44'31" Vv2
1189	8	24	3	27	28" Vv
1209	1	7	47	1	8° Vv; 21°? *in ras.* Py
1229	5	21	30	35	38" Vv; 20°…36" Vv2; 22°24' Pz
1249	10	5	14	9	13" Vv; 4°…11" Vv2; 6°4' Pz; 3°13'20" Py
1	4	9	23	32	
2	9	1	57	39	
3	1	11	21	11	
19	0	4	20	3	30" Py
20	4	13	43	35	
1 day	0	13	10	35	

Validation: The increments in the expanded-year table (129;23,32° for 365 days, 142;34,7° for 366 days) are applied consistently, and yield the 20-year value shown (133;43,35°). In the collected-year table, Vv seems to aim at this increment; in Pz Py, the intended increment is probably 133;43,34°,[17] which has been adopted here. The only value that rests on a consensus (except for gross errors) is that for 1209; this, however, does not fit in the context of Vv, where a 3" is required.

(5.3) Moon, argument.

Vv = Vv,7r (m2 for 1229-1309). Vv2 = Vv,14r (m2 for 1229-1289, where it ends).

1169	5	2	30	51	31'53" Pz; 52'5" Vv2
1189	6	12	17	41	
1209	7	22	4	31	3' Pz; 21° Py
1229	9	1	51	21	50' Vv2
1249	10	11	38	11	=Vv; 37' Pz Py Vv2
1	2	28	43	22	1s Vv
2	6	10	30	38	
3	9	9	14	0	
19	10	11	3	28	
20	1	9	46	50	4° Py; 6' Vv.pc

[17] Mercier (2004, 4) points out that this value is implausible, and emends it to 133;**33**,34°. Some correction in the order of 10' per 20 years is indeed plausible from a comparison with the value in Battani (Nallino 1907, 73), which is 133;35,33°. The excess of about 2' over the emended value is paralleled in the corresponding values for the other planets that have such; in all these cases, the Battani values are in excess of ours by 1'-3'. I will refrain from speculating why.

1 day	0	13	3	54	

Validation: These tables are regular. The increments in the expanded-year table (88;43,22° for 365 days, 101;47,16° for 366 days) are applied consistently, and yield the 20-year value shown (39;46,50°).[18] This is also consistently used in the collected-year table.

(5.4) Node, mean motus.

1169	0	25	42	32	59' Vv
1189	1	22	31	49	46' Vv; 47" Pz Py
1209	2	19	21	6	34' Vv; 4" Pz Py
1229	3	16	8	23	21' Vv; 15°22' Pz; 18°…21" Py
1249	4	12	55	40	13°17' Vv; 11' Pz; 38" Py
1	0	19	19	40	
2	1	8	42	31	
3	1	28	2	11	
19	0	7	27	37	
20	0	26	47	17	
1 day	0	0	3	11	

Validation: These tables are irregular in several ways. In the expanded-year table, the expected increments (of 19;19,40° and 19;22,51°) would entail a 20-year value of 26;**49**,15° instead of the faulty one of 26;**47**,17° that appears in the table. In fact, the tabular increment is in excess by 1" at 8° and at 16°; this may be due to hidden sexagesimals, and the 17" is no doubt intended. More seriously, there is a deficit of 3' at 15° and an excess of 1' at 18°, which account for the total deficit of 2'. In the collected-year table, ms. Vv appears to have intended the incorrect increment of 26;47;17° throughout, though for a radix value that is 17' greater than in Pz Py. The latter two seem to have intended the correct increment of 26;49,17° until 1209, and Py, at least, uses 26;47,17° from then on. I have kept this arrangement.

(5.5) Saturn, mean motus.

Mf: present for collected years 1209-1229 and for expanded years.

1169	11	6	10	14	
1189	7	10	50	16	
1209	3	15	30	18	0" Mf
1229	11	20	10	20	0" Mf; 50" Pz; 10"? Vv
1249	7	24	50	22	55" Pz; 14"? Vv

[18] This value may also be too great. Indeed, Battani (Nallino 1907, 73) has 39;**41**,59°, whereas the values ought to be about equal, since precession is not involved.

1	0	12	13	30	
2	0	24	29	0	30" Mf
3	1	6	42	30	43' Mf
19	7	22	26	30	36' Vv
20	8	4	40	0	
1 day	0	0	2	0	

Validation: The increments in the expanded-year table (12;13,30° and 12;15,30°) are applied consistently, and yield the 20-year value shown (244;40,0°). The increment used in the collected-year table is, however, 244;40,2°, perhaps calculated independently. There are irregularities for 1229 and 1249 in individual manuscripts.

(5.6) Jupiter, mean motus.

1169	1	10	18	57	
1189	9	17	35	22	19s Py.ac; 1° Py
1209	5	24	41	47	34°(*vacat*)'4" Py
1229	2	1	58	12	(*vacat*)'1" Py
1249	10	9	14	37	(*vacat*)' Py; 27" Pz
1	1	0	20	35	26' Py
2	2	0	46	9	
3	3	1	6	44	
19	7	6	55	50	
20	8	7	16	25	
1 day	0	0	4	59	

Validation: The increments in the expanded-year table (30;20,35° and 30;25,34°) are mainly used consistently. The former has a deficit of 10" at year 9, inherited by the 20-year value shown (247;16,25°, where the expected value is 247;16,35°). The value 247;16,25° is used in the collected-year table, except that the increment from 1189 to 1209 is only 247;**6,**25. The scribe of Py may have noticed this, since he left the affected minute values blank.

(5.7) Mars, mean motus.

1169	9	8	0	11	10' Pz.pc Py Vv.?pc
1189	4	26	19	21	
1209	0	14	38	31	28' Py
1229	8	2	57	41	37' Py
1249	3	21	16	51	20°46' Py; 61' Vv
1	6	11	17	6	
2	0	23	5	38	

3	7	4	22	44	41°…8" Py
19	1	7	2	4	
20	7	18	19	10	19° Py

1 day	0	0	31	26	

Validation: These tables are regular. The increments in the expanded-year table (191;17,6° and 191;48,32°) are applied consistently, and yield the 20-year value shown (228,19,10°). This is also consistently used in the collected-year table.

(5.8) Venus, argument.

1169	8	3	13	43	7s19° Py
1189	2	6	57	2	1s22° Py
1209	8	10	40	20	7s24° Py; 30" Py
1229	2	14	23	38	8" Pz; 0° Py
1249	8	18	6	56	7' Pz; 4° Py

1	7	15	1	58	(*vacat*)° Py
2	3	0	39	56	3' Pz.pc Py; <.>9 Pz.ac
3	10	15	41	54	4' Py
19	10	18	41	21	
20	6	3	43	18	

1 day	0	0	37	0	

Validation: The expanded-year table is irregular. Indeed, the expected increments (of 225;1,58° and 225;39,58°) would entail a 20-year value of 183;**44**,20° instead of the faulty one of 183;**43**,18° that appears in the table. It turns out that the tabular increment is in deficit by 1" at 9° and at 20°, which may be due to hidden sexagesimals, so the 18" are likely to be intended. In minute values, there are deficits of 1' at 2°, 12° and 16° and an excess of 2' at 14°, accounting for the total deficit of 1'. In the collected-year table, the faulty 20-year value has been applied consistently, except that the increment from 1169 to 1189 appears to be 1" too great. For 1169-1209, Py seems to have its own radix value, but Py is altogether very faulty.

(5.9) Mercury, argument.

1169	4	2	17	50	
1189	4	16	46	40	
1209	5	1	15	30	
1229	5	15	44	20	43'35" Vv
1249	6	0	13	10	1s…11'40" Vv

1	1	23	56	50	
2	3	21	0	6	50' *omnes*

3	5	14	56	56	
19	10	20	32	0	
20	0	14	28	50	5" Vv (cf. 1229, 1249)
1 day	0	3	6	24	

Validation: In the expanded-year table, the 365-day increment (53;56,50°) is normal, whereas the 366-day increment (57;3,16°) is 2" greater than expected from the values for 1 year and 1 day. As this is applied 5 times in the table, the 20-year value (14;28,50°) becomes 10" greater than expectation. In the collected-year table, this value is applied consistently by Pz and Py. In Vv there is a gross error in the 20-year value, and the secondary hand in Vv has used this faulty value for calculating the values for 1229 and 1249.

For a rough and easy sanity check, I have compared the mean positions for year "1169" elapsed (i.e., AD 1170, March 1, noon) to those obtained from the Toulouse tables[19] for "1152"+"17" elapsed. To the Toulouse values I have added the motion during 1 day, since, e.g., Monday starts at noon on Sunday in the Toulouse tables but at noon on Monday in our tables. I have also subtracted the motion during 0;53h from the Toulouse values, to take some account of the longitude difference between Toulouse and Pisa.[20] When this is done, the values in our tables turn out to be about 8° greater than the Toulouse values (give or take about 1°) for the Sun, the lunar motus, the node (changing its sign), Saturn, Jupiter and Mars.[21] This is chiefly because the velocities in our table include precession (cf. "motum octave (sphere)" in the passage quoted under (2) above) whereas the Toulouse velocities do not. The precession of about 8° more or less fits the common precession rates (51" or 54" per year) if it is assumed to have been zero at about the time of the Hijra (AD 622). I do not know whether this is by accident.

In the three cases where precession is left out of account, namely, the lunar argument, Venus and Mercury, the Toulouse mean positions are indeed about the same as ours (again, give or take about 1°).[22] Here and above, these fluctuations are no doubt mainly due to small differences in radix val-

[19] See Poulle 1994 and F.S. Pedersen 2002, 1197 ff.

[20] In Ptolemy's Geography, the longitudes from the West are 20;10° for Toulouse and 33;30° for Pisa. I do not know of any medieval longitude value for Toulouse that can be of use in connection with the 33° value for Pisa.

[21] The six differences in question lie between 6;37° (Saturn) and 9;17° (lunar motus) with an average of 8;9°. The precession between AD 622 and 1169 might be 7;45° (for a rate of 51"/year) or 8;17° (for a rate of 1°/66 years).

[22] The three differences in question lie between -0;49° (Venus) and +1;17° (lunar argument) with an average of +0;9°.

ues and velocities between the two sets of tables, including but not limited to faulty velocity parameters in our tables, as mentioned above. But on the whole, there seem to be no gross errors in the values we have.

From the fact that the collected-year table in Vv originally ended in 1209, and from the fact that most of the variant values in all manuscripts may be located to the years 1229 and 1249 (cf., e.g., the table for the lunar node), it seems safe to conclude that the original tables ended in 1209 at the latest, and that the later values were computed independently on several different occasions.

(6) The mean motion tables in ms. K.

I quote the collected-year tables in full. Other values are only reproduced if they show peculiarities relative to the mainstream values established above.

As in most cases, it cannot be said what part of the rest of the tradition K relates to. I underscore the values that seem to bear no relation to those known from the manuscripts quoted above.

(6.1, 183v) Sun, mean motus, whole of collected-year table.

	s	°	'	"
1169	11	15	33	20
1189	11	15	42	20
1209	11	15	51	20
1229	11	15	15	21
1249	11	15	24	21

(6.2, 184r) Moon, mean motus.

1169	8	10	19	53	
1189	9	24	3	28	
1209	1	7	47	3	3": fits normal tabular increment of 4s13;43,35°
1229	5	11	23	18	
1249	9	24	66	54	increment = 4s13;43,**36**°

(6.3, 184r) Moon, argument.

1169	5	2	31	53	
1189	6	12	18	43	
1209	7	22	5	33	
1229	8	22	59	47	
1249	9	32	5	97	100' carried as 1°

(6.4, 185v) Node, mean motus.

1169	0	25	42	32
1189	1	22	31	49
1209	1	19	21	6

1229	3	16	7	59	
1249	3	42	56	96	
1	0	19	19	40	
20	0	26	49	37	

(6.5, 185v) Saturn, mean motus.

1169	11	6	10	13	
1189	7	19	15	13	
1209	3	15	30	20	
1229	11	20	8	43	
1249	9	24	48	43	9s: erroneous, intended value unclear

(6.6, 186v) Jupiter, mean motus.

1169	1	10	18	57	
1189	9	7	35	22	
1209	5	24	51	47	
1229	2	21	36	9	
1249	9	31	40	9	Uses the faulty 20-year value from below.
1	0	29	0	12	The 1-year increment is constant throughout this table, and no account is taken of leap-years.
20	7	10	4	0	

(6.7, 187v) Mars, mean motus.

1169	9	8	0	11
1189	4	26	29	31
1209	0	14	38	51
1229	8	2	35	45
1249	15	20	54	65
1	6	11	17	6
20	7	18	19	20

(6.8, 188v) Venus, argument.

1169	8	3	13	43
1189	2	6	57	1
1209	8	10	40	19
1229	2	23	55	40
1249	8	26	98	58

(6.9, 189v) Mercury, argument.

1169	4	2	17	50
1189	4	16	46	40
1209	5	1	15	30
1229	5	13	23	30
1249	5	27	51	80

(7) Some characteristics of the mean motion tables of ms. K.

(a) The values for 1169, 1189 and 1209 are the same as in the Pisa tables attributed to Abraham b. Ezra, except that those for the lunar argument are systematically 0;1,2° greater than standard, for no obvious reason. The quality of the text is average, with quite a few errors; on the other hand, K shows some readings that may be original (e.g., Moon, mean motus for 1209, 3''; Mars, mean motus for 1169, 0'). As a source for these values, then, K may be largely independent of the other witnesses examined.

(b) The values for 1229 and 1249 are in the same hand as the earlier ones, but they are not the same as in the standard tables. Apart from one or two errors, they differ between themselves by the amount of the 20-year value apparent from K, whether this is faulty or not (cf. the Jupiter table for a particularly bad case). The 1229 values seem to be independently calculated;[23] perhaps this calculation is meant to constitute the "transferral to Paris" mentioned in the rules that were quoted in (1) above. The 1249 values were probably made by yet another person, who was quite an ignoramus, since he seems to have believed that 1 degree consists of 100 minutes (cf. Moon, argument, 1249). For this reason there are mostly no carries in the 1249 values, making many of the sexagesimals nonsensical. The 1229 calculator, whatever his assumptions were, did not at least make such gross mistakes.

(c) The common origin of all the manuscripts of the Pisa tables was probably a copy where the collected-year tables ended in 1209 or earlier. Indeed, 1209 is where ms. Vv ended originally; and the textual variants suggest that there were several independent attempts to continue these tables for 1229 and later. Ms. K seems to be the most conspicuous instance of these attempts.

[23] On the face of it, they are valid for times that are some 16-19 hours earlier than the times for the mainstream 1229 values quoted earlier. Thus, e.g., the value in ms. K for the lunar mean motus, 161;23,18°, is less than the mainstream 171;30,35° by 10;7,17°, which corresponds to the lunar mean motus during about 18;28h. The analogous time differences are: Sun, 18;16h; Moon, argument, 16;16h; Saturn, 19;30h; Mars, 16;44h; Venus, 18;28h (if the 23° in the K value is emended into 13°); Mercury, 18;8h. I do not know what to make of this apparent regularity; to judge by its size, the difference does not make sense as an attempt at adapting for a meridian different from that of Pisa. In any case the time difference for the lunar node is only 3h (negative since the node is retrograde), and there seems to be no way to make the Jupiter value follow the scheme.

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